REFLECTIONS OF DEVELOPMENTS IN EDUCATIONAL TECHNIQUES IN THE DESIGN OF A NEW TEXTBOOK ON DESCRIPTIVE GEOMETRY

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The article discusses innovative methods in modernization of teaching descriptive geometry at the Faculty of Mathematics and Physics at Charles University in Prague. Our goal is to increase interest in studying classical and descriptive geometry primarily through 3D computer modelling. I have been seeking to establish a stronger connection between descriptive geometry and its practical application and the extension of descriptive geometry with knowledge of computer graphics and computer geometry. The integration of descriptive geometry with 3D computer modelling appears to follow as a logical step. In order to provide insight into more complex geometric problems and to increase the interest in geometry, I have integrated 3D computer modelling in my descriptive geometry lessons. I plan to use outputs from 3D computer modelling software into my new textbook on descriptive geometry for undergraduate students.

Keywords: descriptive geometry; 3D computer modelling; textbook on descriptive geometry

MOTIVATION AND PRACTICAL APPLICATION

Descriptive geometry (Paré et al., 1996; Pottmann et al., 2007; Robertson, 1966) represents an important area of classical geometry dealing with the representation of three-dimensional objects in two dimensions where 3D computer modelling and interactive software visualization can be applied with potentially significant impacts. Hence, the typical task in descriptive geometry is to represent three-dimensional objects on a two-dimensional display planar surface and to reconstruct 3D objects from the two-dimensional result of the projection. Descriptive geometry deals with those representations which are one-to-one correspondent. In order to gain deep understanding of descriptive geometry it is necessary to have knowledge of the fundamentals of geometry, the properties of geometrical objects in the plane and in the space, and their relations. This means that, in addition to geometrical projection, descriptive geometry should focus on special types of technically important curves and surfaces in engineering practice.

In general, geometry can be conceived as an independent discipline comprising various branches and it also forms the basis for many modern applications. The motivation for studying geometry can be found in building practice, engineering and construction practice, architectural and industrial design, production industries, export of real interiors and exteriors into the virtual worlds of computer games, digitization of real objects by 3D scanning, digital surface reconstruction from point clouds, replication of the shapes of real objects using 3D printing, computer graphics and many more, (Pottmann et al., 2007). The common basis of all these modern applications is the combination of geometric principles and knowledge. Applied methods are often based on elementary geometry.

The role especially of descriptive geometry in practice is irreplaceable in such branches in which correct visualization is crucial. All of the mentioned application fields are dependent on clear

illustrations and visualization. Overall, geometry in the plane and in the space, i.e. the properties of geometrical objects and their relations, form a part of many modern and contemporary scientific fields.

Geometry represents one of the highly demanding fields of mathematical science which require logical thinking and which also strongly stimulates spatial imagination, (Hilbert, 1999). The study of geometry, and especially descriptive geometry, represents an ongoing challenge in terms of research and practice.

On the top level, the paper is organized into two parts. The first part explains possible novel methods of teaching descriptive geometry which include 3D computer modelling and interactive software visualization. This part is largely a summary of the existing concepts used in my lessons. The second part contains the main contribution of my work: the description of the upcoming textbook on descriptive geometry for undergraduate students. The conclusion is devoted to my future work and research in geometric fields.

MODERNIZATION OF TEACHING DESCRIPTIVE AND CLASSICAL GEOMETRY WITH 3D COMPUTER MODELLING

There exist professional graphics software applications and environments which provide the required user input tools, and speed up production and are commonly used in the process of designing, design documentation and construction for modelling and drawing, and generally throughout the entire design process (Farin et al., 2002).

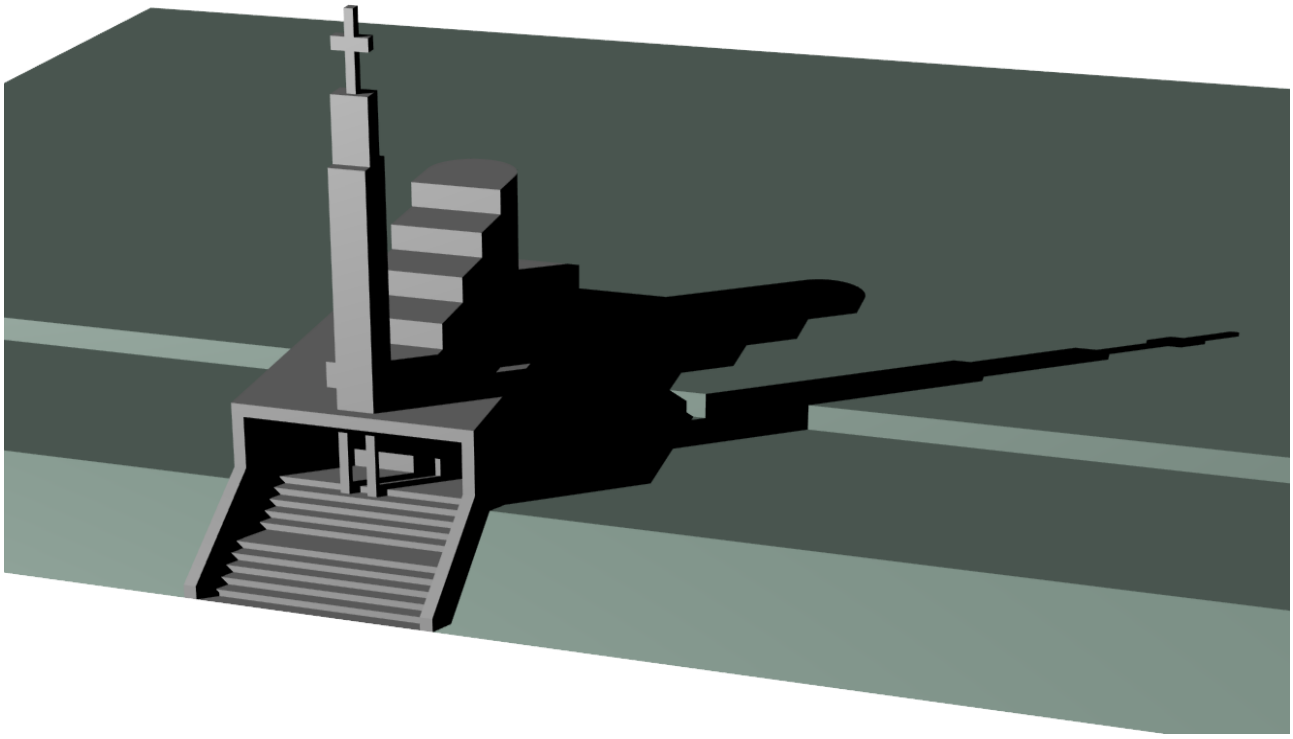

Figure 1: Example of 3D model created in Rhinoceros – the spatial situation of geometry of shadows on real object.

Similar software can be used in teaching traditional geometric subjects, including descriptive geometry. I have integrated 3D computer modelling in my descriptive geometry lessons at the Faculty of Mathematics and Physics at Charles University in Prague. I work mainly with the *Rhinoceros* (*NURBS Modelling for Windows*) software which is a commercial NURBS-based 3D modelling tool, (McNeel, 1999), commonly used in the process of designing, design documentation and construction. I use Rhinoceros to create 3D models of geometric objects and situations in the space, (Surynková, 2013). It should also be noted that if we work with 3D modelling software, we can change the view of a designed object and see spatial geometric objects from another perspective which provides a clearer idea of the object. Example of spatial 3D model is provided in Figure 1. We also use Rhinoceros to draw up constructions in the plane. Example of computer drawing is shown in Figure 2.

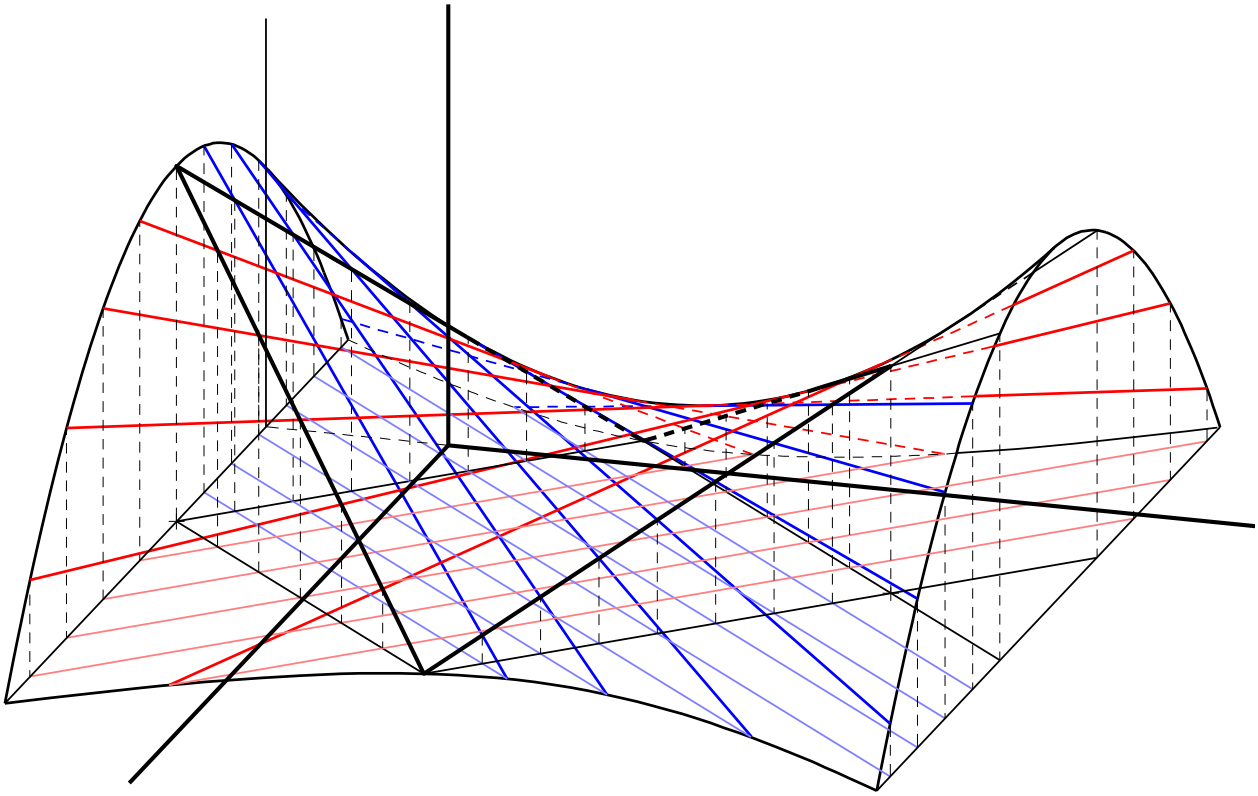

Figure 2: Example of computer drawing created in Rhinoceros – an orthogonal axonometric projection of hyperbolic paraboloid.

The study of descriptive geometry includes, both at secondary schools and colleges, sketching and drawing activities. We do not intend to abandon traditional hand drawing methods because computer drafting is not efficient in developing our skill and thoroughness. Drawing and sketching helps us develop our precision skills and patience and we rely on these tools when developing of our initial ideas and finding solutions to geometrical problems. Computer drafting is a modern auxiliary method which is also capable of yielding more precise results. The examples of hand drawings are provided in Figure 3.

Thus, I combine the both approaches to the teaching of descriptive geometry - the traditional descriptive geometry teaching methods and procedures (sketching and drawing activities) and

modern computer-based experiments with digital modelling tools. As has already been pointed out, I use 3D computer modelling to create 3D models of geometric objects and situations in the space which can help my students understand geometrical problems in intuitive and natural way. I use these outputs during my lessons as illustrations of geometrical properties of studied objects. Moreover I show geometrical constructions in the plane and in the space using graphical software tools so that students can discover principles and proofs of geometrical theorems more easily.

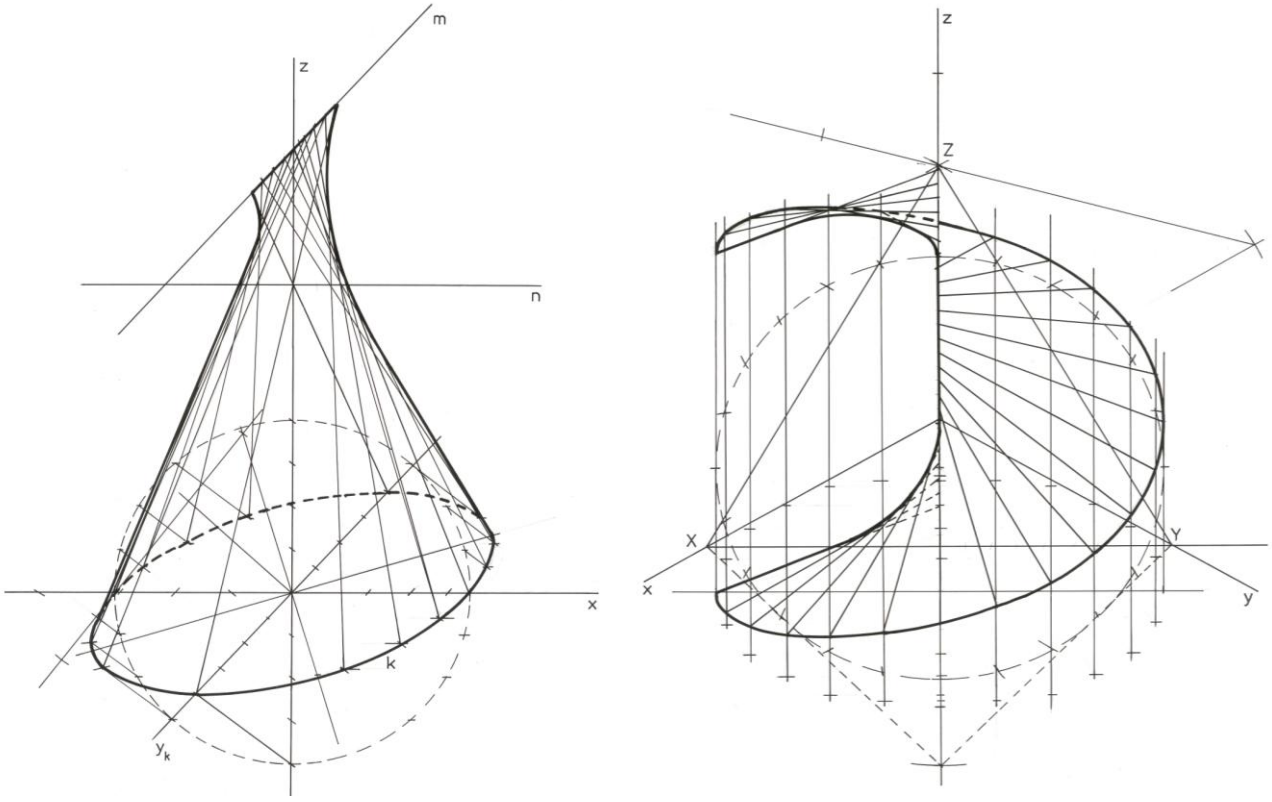

Figure 3: Examples of hand drawings – a parallel projection of ruled surfaces.

Students meet with 3D computer modelling within compulsory lessons of descriptive geometry and also can attend the seminars of applied descriptive geometry where practical applications of descriptive geometry and 3D modelling are mentioned and discussed. Students also use practically 3D modelling software during these lessons and seminars and can create themselves the outputs - 3D computer models and planar constructions.

It is not necessary to work only with Rhinoceros or with expensive CAD applications, which are common commercial 3D modelling tools used for computer aided design (*CAD*). As there is a wide range of inexpensive or free software applications for geometry and mathematics, students and teachers can use them. One of the most widespread free geometrical tools is a mathematics and geometry dynamic software GeoGebra. I use GeoGebra to create planar and spatial constructions and to demonstrate the proofs of geometrical theorems, (Surynková, 2014). Example of planar construction created in GeoGebra is provided in Figure 4. The great advantage of GeoGebra is the possibility to change dynamically the parameters of the designed geometrical objects. My students create their homework or seminar project using GeoGebra for example.

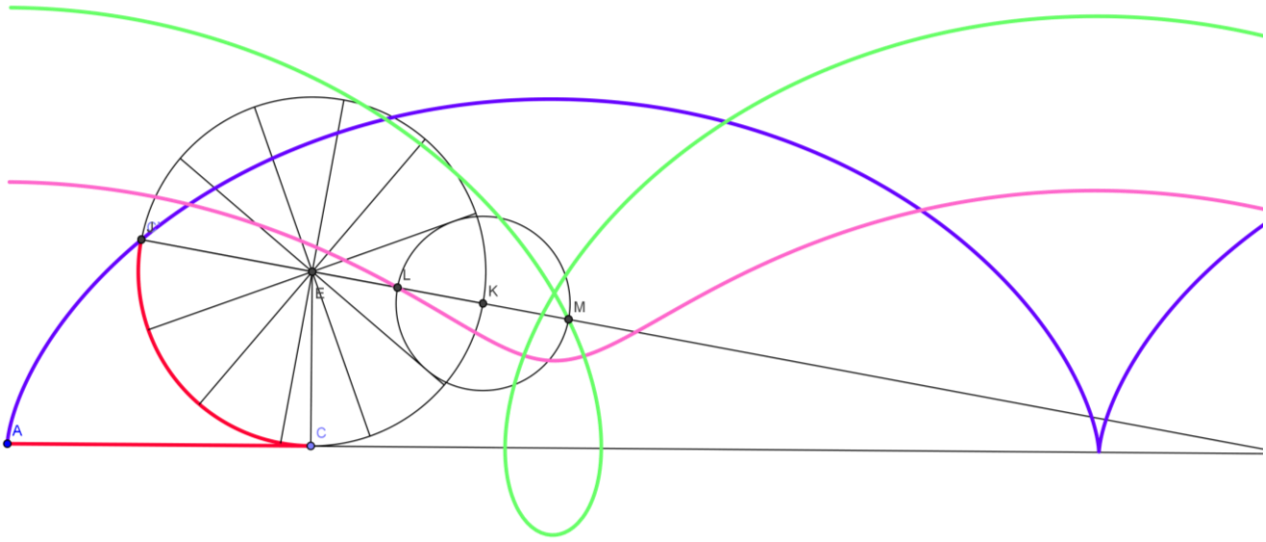

Figure 4: Example of planar construction created in GeoGebra - the paths (cycloids) of points which are obtained by rolling the circle on the straight line.

I have been gathering all of the aforementioned outputs from 3D computer modelling and computer drawings obtained during the preparation of descriptive geometry lessons to create electronic collections of examples as well as for the purposes of new electronic methods of study of materials that relate to various geometric topics. All of these outputs are published on the website http://www.surynkova.info/, (Surynková, 2015). The site is continuously updated and it is intended not only for my students but also for everybody who is interested in geometry (some of the links are in English). New study materials and examples are dedicated to geometric constructions; there are also 3D computer models, examples of students' works and many more.

Visualization of the geometric constructions, 3D models of geometric objects, situations in the space and 3D computer modelling and modern digital tools in general can be used to improve the teaching of geometry. 3D computer modelling strongly stimulates spatial imagination and helps students to understand geometric concepts practically. As it has been shown in my previous teaching experiments, I also find this approach very valuable from the practical point of view as it can demonstrate young students that today's information age practice can be nicely integrated with classroom teaching. Using computer software in classrooms prepares students for their future profession at the same time.

THE UPCOMING TEXTBOOK ON DESCRIPTIVE GEOMETRY DESIGNED WITH 3D COMPUTER MODELS

The use of modelling and graphics software in teaching geometry increases students' interest in the subject and ensures their active involvement in the lessons, which is evident from the reactions of my students and also from their interest in these issues when dealing with their seminar projects or bachelor and master theses. 3D computer modelling is also an efficient aid in innovating the teaching of geometry and achieving better results.

I have been seeking to establish a stronger connection between descriptive geometry and its practical application and the extension of descriptive geometry with knowledge of computer

graphics and computer geometry. The integration of descriptive geometry with 3D computer modelling appears to follow as a logical step.

This paper explicitly addresses the content and the design of a new printed textbook on descriptive geometry which I have been working on. This textbook is primarily dedicated to geometric topics such as curves and surfaces, solids, their definitions and properties, their parallel and central projections and the geometry of shadows. The textbook is intended mainly for students of the Faculty of Mathematics and Physics of Charles University in Prague and the first edition is planned to be published in Czech. The textbook will be illustrated using 3D computer modelling and modern software visualizations. The important part of the publication is the collection of examples with solutions and examples for testing purposes.

Case study for textbook chapters

Let us now focus on some parts of the planned chapters in the upcoming book, and describe its expected design. I am currently working on the theoretical aspects of special groups of surfaces used in engineering practice. The book will define each regular geometric surface and introduce its properties. Let us, for instance, look at an example of a part dedicated to helical surfaces; the concept of the chapter is as follows.

First, a theoretical explication regarding the determination of helical surfaces is provided, accompanied with illustrations from the 3D computer modelling software. It is assumed that the source files of most pictures from the textbook are available on the attached removable media to allow practical exercises regarding the properties of the discussed surface or spatial situations directly in the software. The illustration of helical surfaces with a brief description is shown in Figure 5.

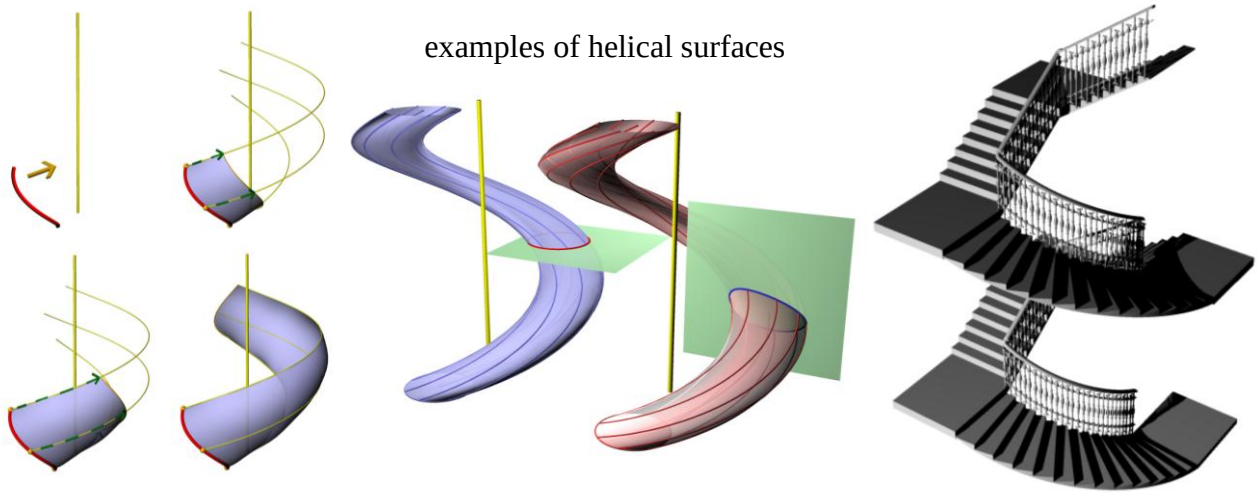

Figure 5: The illustration of helical surfaces.

The second part of each chapter is devoted to parallel and central projections of the studied surfaces, accompanied with a typical example including a detailed step-by-step solution and illustration. The typical task is to construct a parallel or perspective view (a two-dimensional image) of a particular

surface. Figure 6 shows an orthogonal axonometric projection and central projection of a helical surface, defined by an axis and screwing segment line. The result of the projection and also the situation in the space are visible. Every illustration is made using 3D computer modelling.

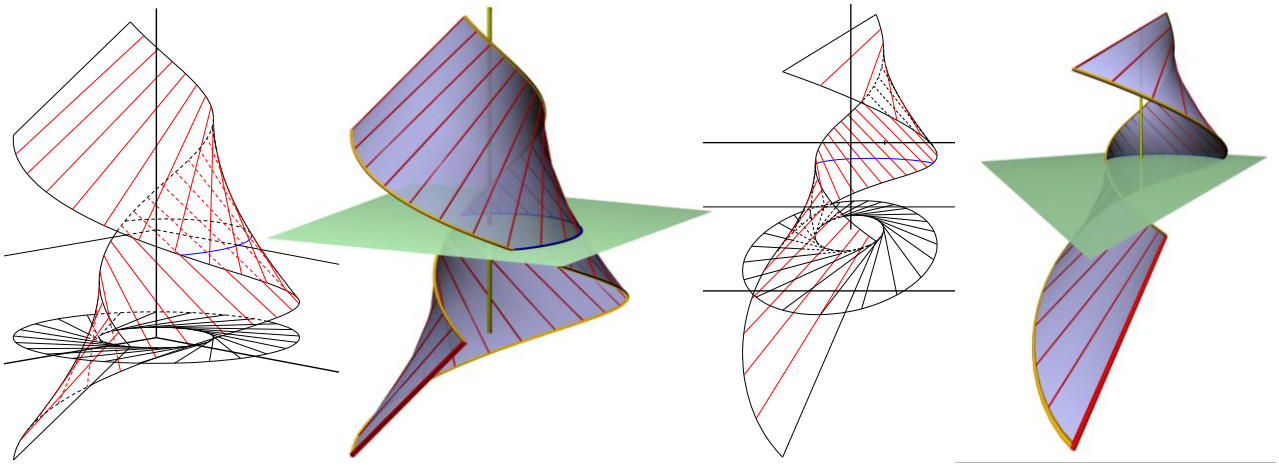

Figure 6: The results of the projections of helical surface and the situations in the space – an orthogonal axonometric projection (left) and a central projection (right) of the same helical surface.

The last part of every chapter comprises a collection of examples for exercising the properties of surfaces in various projections. Students can solve the tasks using 3D modelling or graphics software or they can draw the solutions by hand. When using software, it is necessary to construct the silhouette of the surface; if drawn by hand, the aim is to depict some of the important curves on the surface. In both cases, the result is a planar image.

An interesting additional feature of these examples is the possibility to model the surfaces in 3D modelling computer software in space. The spatial situation and principles of projection can also be demonstrated. The virtual model of the spatial situation and 3D virtual models of surfaces make a significant contribution to the development of spatial imagination. Some examples in the book are added in the form of 3D models on attached removable media and additional 3D computer models can be created in cooperation with my students, for example, as part of their theses.

CONCLUSION AND FUTURE WORK

Two areas are addressed in this paper – the possible *methods of innovation in teaching descriptive geometry* including 3D computer modelling and the creation of study materials and web support for descriptive geometry and *the description of the upcoming textbook on descriptive geometry* for undergraduate students.

The main aim is to improve and innovate the methods of teaching descriptive geometry by using 3D computer modelling and enabling connection with practice. It is planned to integrate the suggested outputs from 3D computer modelling software into my new textbook on descriptive geometry. In the future, it is envisaged to publish the textbook in English translation. Some parts of the textbook are also planned to be published on the Internet.

For the future work I am also considering to improve teaching of descriptive geometry also in other ways. It seems to be promising to intensify the extension of descriptive geometry with knowledge of computer graphics and computer modelling. Each regular geometric surface (and also auxiliary

curves) in my new textbook will be described using mathematical equations. Then we can model these geometric objects in mathematical software.

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