3D Modeling in Teaching and Learning Geometry

Petra Surynková

Charles University in Prague, Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Praha 8, Czech Republic

This article addresses the application of computer modeling in teaching and learning geometry. Our aim is to increase the interest of students in studying geometry at secondary schools and colleges. The main field of our interest is study of synthetic and descriptive geometry – geometric constructions, projections, geometry of curves and surfaces. One possible approach of improvement in studying geometry is the integration of computer software in the teaching process. Here we will demonstrate the usage of Rhinoceros program (Rhino) for visualization, for proving geometric problems in the plane and in the space or for the demonstration practical uses of geometry. Nowadays we have a great collection of pictures which illustrates examples of surfaces in building practice and architecture. We show the usage of software on some concrete examples. The outputs can be used in publications and also for home schooling and e-learning.

Introduction and motivation

Geometry, the study of properties and relations of geometric figures, is an important and essential branch of mathematics. Geometry can be conceived as an independent discipline with many branches – Euclidean geometry, differential geometry, algebraic geometry, topology, no-Euclidian geometry and so on. Geometry is useful for learning other branches of mathematics and it can also be used in a wide range of scientific and technical disciplines. Some scientific branches require direct knowledge of geometry.

The main field of our interest is the study of classical geometry and descriptive geometry – geometric constructions, projections, geometry of curves and surfaces. Mainly it is geometry which allows the representation of three dimensional objects in two dimensions. Descriptive geometry which studies the properties of projections has become the language of designers and engineers. Classical geometry is geometry of the Euclidean plane and space. Our aim is to show practical uses of this geometry. Mainly we will focus on examples of using geometry of surfaces.

Geometry is important for everyone, not only for technicians, designers, architects, builders or civil engineers. We all need good visual imagination in our everyday life as well. The two and three dimensional shapes which surround us are originated in geometry. The world we live in is influenced by geometry. If we know how to understand and apply the relationship between shapes and sizes we can use it more efficiently. Some people think in images and shapes so they need the understanding of geometry to be able to do that.

Without the use of geometry the great works of artists, painters and builders would only have stayed in ideas and dreams.

According to (Hilbert, 1999) the study of geometry develops logical reasoning and deductive thinking which helps us expand both mentally and mathematically. If we learn to use geometry we also learn to think logically. It's very important in everyday life – many difficult problems can be erased and the simple solutions can be found. Students can often solve problems from other fields more easily if they represent the problems geometrically.

The rest of this paper is organized as follows: the next section "The study of geometry" is devoted to current problems of the unpopularity and the difficulty of studying geometry. "How to increase the interest of students in studying classical geometry; use of computers in the teaching process" is the main subject of the following section. Finally in the section "The examples of 3D modeling" we will demonstrate the advantages of modeling computer systems especially Rhinoceros on concrete examples from the field of descriptive geometry and geometry of surfaces. The models which illustrate examples of surfaces in building practice and architecture will be shown. In conclusion we will discuss the advantages of geometric modeling systems and the responses from our undergraduate students.

The study of geometry

The study of geometry can be very difficult. This branch of mathematics isn't popular among students; see for example (Schwartz, 2008a; Schwartz, 2008b). Drawings (the results of geometric projections of some 3D model) are sometimes very difficult to understand. For that reason geometric problems must be provided with clear examples. Intuitive understanding plays a major role in geometry. With the aid of spatial imagination we can illuminate the problems of geometry. It is possible in many cases to show the geometric outline of the methods of investigation and proof without entering needlessly into details. The problem can be more understandable without strict definitions and actual calculations. Such intuition has a great value not only for research workers, but also for anyone who wishes to study and appreciate the results of research in geometry. Of course, if we understand the main principles of a problem then we can use exact definitions.

The currently predominant view among students and the general public is that classical geometry is not important and useful. Drawings of classical geometry can be replaced by the outputs of modern computer software. Of course, computers can help us solve geometric problems and increase the efficiency of our work but we still have to know the basic principles and rules in geometry.

Is it possible to learn geometry? Yes, but it would be easier for students if they had encountered classical geometry, constructions and geometric proofs earlier. Sometimes students of technical specializations experience geometry only at college. That is too late. We work mainly with undergraduate students, so what can be done to make college geometry more comprehensible? How to increase the interest of students in studying classical geometry at secondary schools and colleges? This is the main subject of this article.

How to increase the interest in studying geometry; use of computers in the teaching process

Our aim is to increase the interest of students in studying classical geometry at secondary schools and colleges. One possible approach of improvement in studying geometry is the integration of computer software in the teaching process. This way seems to be interesting, attractive and motivational for students. Indeed the usage of computers in education is very current. Computers influence our everyday life including geometry. We have to follow the general trend.

Nowadays, computer-aided design (CAD) is commonly used in the process of design, design documentation, construction and manufacturing processes. There exist a wide range of software and environments which provide the user input tools for modeling, drawing, documentation and design process. These software and environments can be used to design curves and geometric objects in the plane and curves, surfaces and solids in the space. According to the applications more than just shapes can be involved. In modern modeling software we can also work with rotations and other transformations; we can change the view of a designed object. Some software provides dynamic modeling. Technical and engineering drawings must contain material information and the methods of construction. Computer-aided design is used in numerous fields: industry, engineering, science and many others. The particular use varies according to the profession of the user and the type of software.

These modern methods which are widespread in various branches can be useful in the teaching process, too. We can prepare students for their future employment. We do not advocate the usage of computers at any price. Of course we still place emphasis on the understanding of the principles used in geometry. Sometimes it is of significant importance to work without technical support and use only our own mind.

I have my own experience of teaching classical, descriptive, and computational geometry at universities – Charles University in Prague – Faculty of Mathematics and Physics and Czech Technical University in Prague – Faculty of Architecture. College mathematics and geometry is very difficult for many students. It is necessary to motivate and to arouse their interest in geometry. As was mentioned above, it is necessary to improve the teaching of geometry at elementary schools and at secondary schools.

In my lessons I use computer software for visualization, for the proving of geometric problems in the plane and in the space, for the demonstration of the application of geometry in practice, for the creations of study materials for home schooling and e-learning or for the transformation geometric problems into mathematical form. I work mainly with Rhinoceros - NURBS modeling for Windows (Rhino), Cabri II Plus, Cabri 3D, MATLAB, Maple and with GeoGebra. Use by teachers and students is always free of charge, it is the great advantage of GeoGebra. Consequently it can be used by students for home schooling and e-learning. Other software is available for teachers and students at our school. Nowadays we have extensive database of geometric tasks, images and 3D models – the outputs of these software.

I use these software for creation of stepwise guides through geometric construction which can help my students understand the problem in intuitive and natural way. Moreover I show special constructions applied in descriptive geometry and due to included functions and tools in these software students can discover proofs more easily. In this contribution we will demonstrate the advantages of the use of geometric modeling systems on examples from the field of descriptive geometry. Several examples of constructions used in descriptive geometry will be shown, too. Good geometric imagination and perception is very important for understanding spatial constructions in geometry. It is not possible to memorize the constructions; we have to understand geometric problems. Classical geometry is very useful branch of mathematics in practical applications. Geometry of surfaces, the main field of our interest in this contribution, is very important in building practice and architecture. The models which illustrate examples of the usage of special surfaces will be shown.

The examples of 3D modeling

The aim of this paper is to show applications of classical and descriptive geometry especially geometry of surfaces. See the following examples of geometric surfaces used in architecture. These figures were made in Rhinoceros - NURBS modeling for Windows and I use these outputs as study materials in my lessons at Charles University in Prague and Czech Technical University in Prague. The outputs can be also used for home schooling and e-learning. Rhinoceros (Rhino) is commercial NURBS-based 3D modeling tool. This software is commonly used in the process of design, design documentation or construction. Rhinoceros has many tools and functions for graphics designers. Rhinoceros can create, edit, analyze, document, render, animate NURBS curves, surfaces, and solids.

If we want to work with similar computer modeling software (it is not necessary to work just with Rhino), we have to know geometric principles not only the special tools and functions which that software provides. On one hand these software are useful aid in the popularization of geometry in general but on the other hand we need to understand geometry if we work with them. Mainly mathematical and modeling computer software can motivate our students to discover the beauty of geometry. Of course, then students have to learn geometry in classical way. Finally students can create the outputs with some software themselves and apply their knowledge gained from studying. The construction of the models is not just a computer amusement. See the figures – the demonstration of the usage of Rhino. Theory of these surfaces can be found in (Pottman et al, 2007), (Farin, 2002), (Gerald et al., 2002).

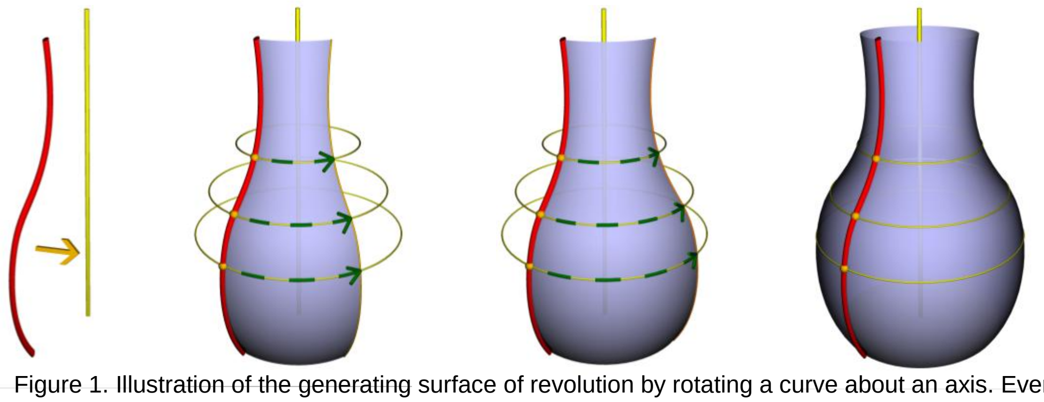

Figure 1. Illustration of the generating surface of revolution by rotating a curve about an axis. Every point of the curve describes a circle with the centre on the axis.

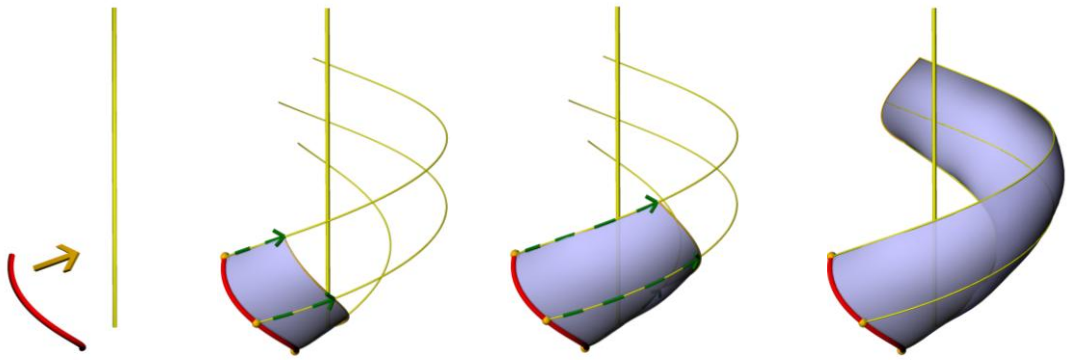

Figure 2. Illustration of the generating helical surfaces by helical motion a curve about an axis. Every point of the curve describes a helix with the same axis as the axis of surface.

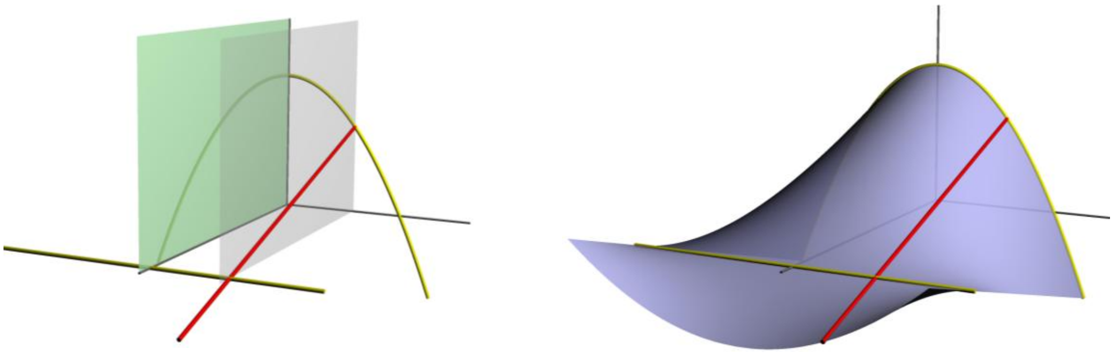

Figure 3. Illustration of the generating ruled surfaces by a moving straight line. This is the special type of ruled surface which is called a parabolic conoid. Straight lines on the surface are parallel with the depicted plane (green) and intersect parabola and straight line (yellow). On the left input elements, on the right the final shape of the conoid.

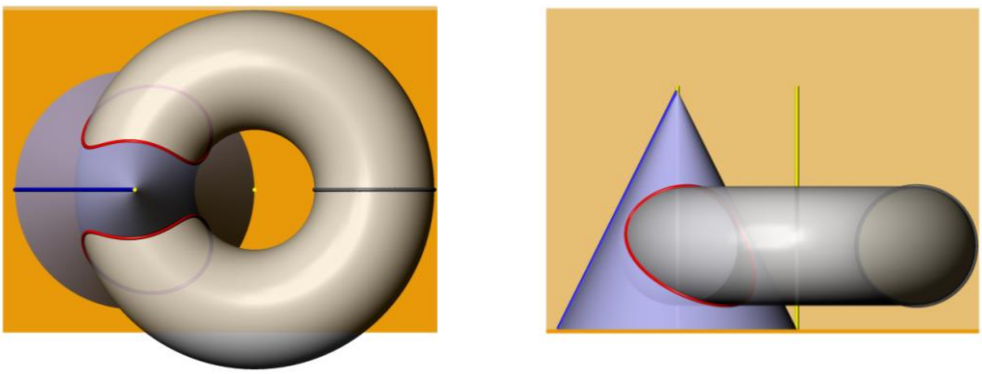

Figure 4. Intersection curves of two surfaces of revolution; on the left top view, on the right front view.

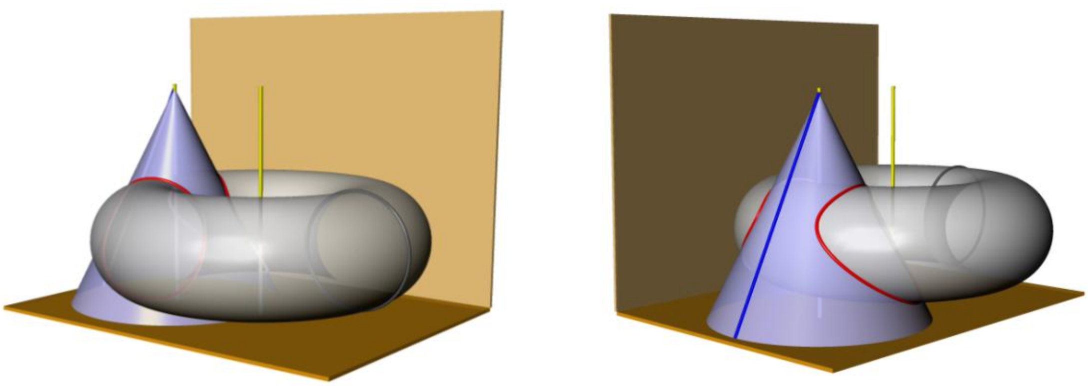

Figure 5. Perspective projection of intersection curves of two surfaces of revolution. The use of appropriate perspective or parallel views to illustrate spatial situations has the main advantage that we can imagine spatial situation in clearly way.

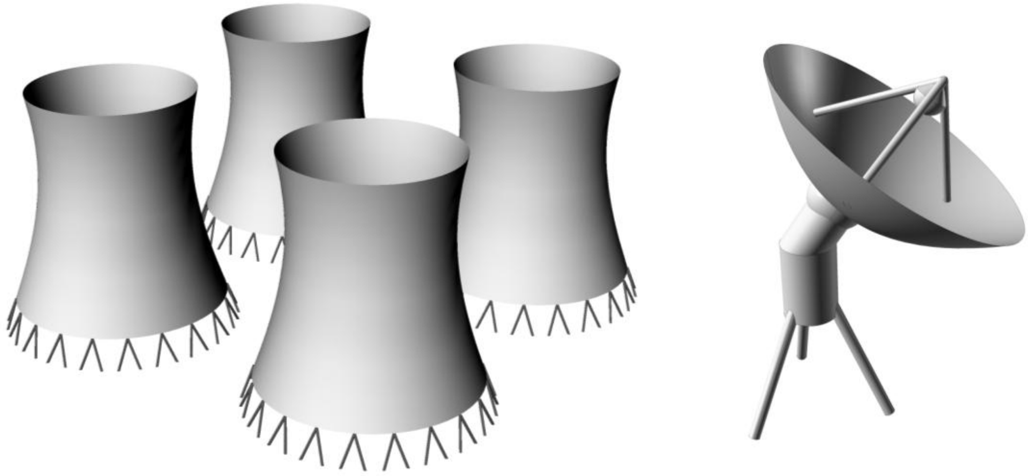

Figure 6. Examples of practical uses of surfaces of revolution; on the left one-sheet rotational hyperboloids as cooling towers, on the right rotational paraboloid as radiotelescope.

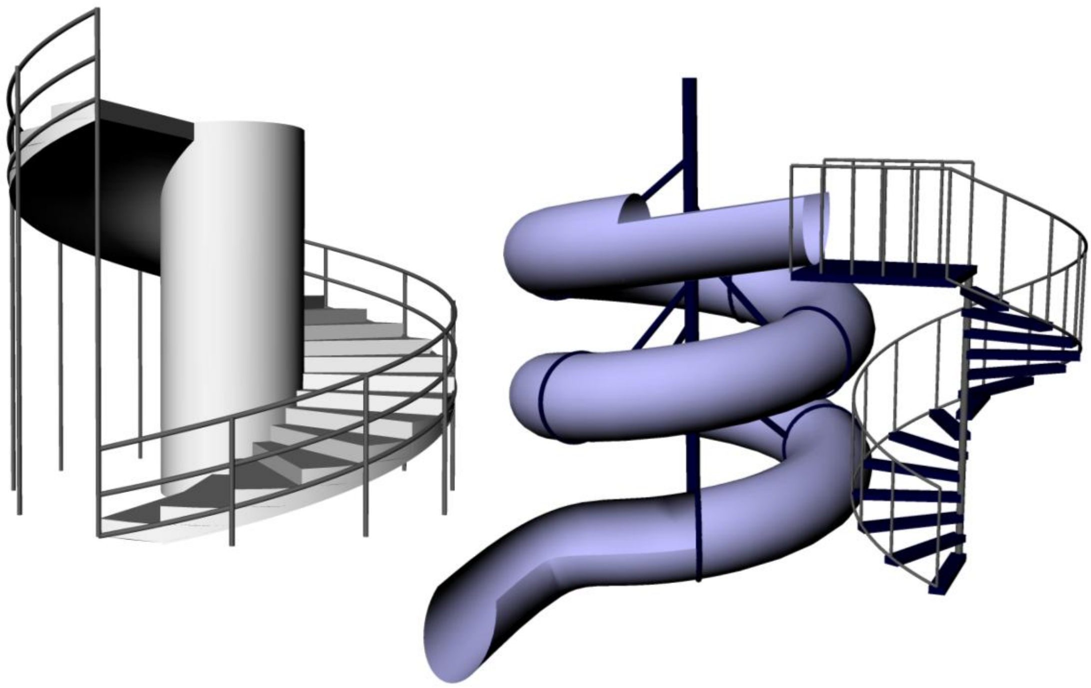

Figure 7. Examples of practical uses of helical surfaces.

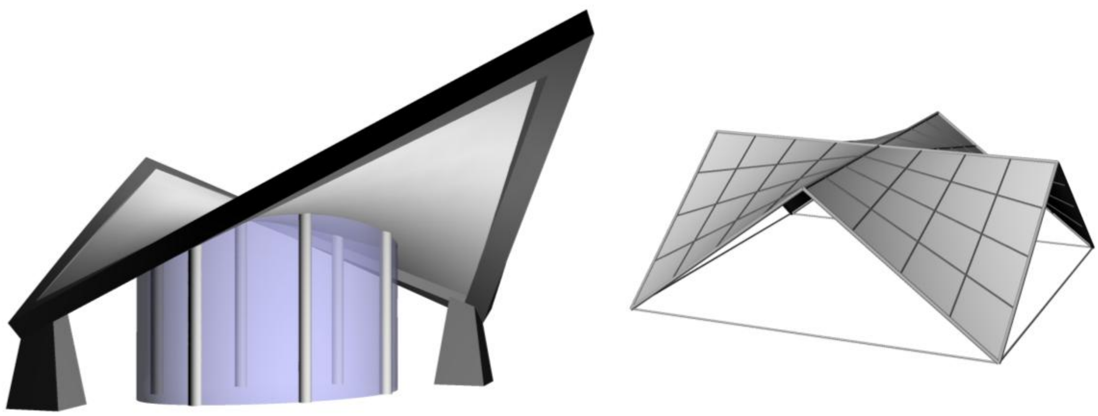

Figure 8. Examples of hyperbolic paraboloids in practical use.

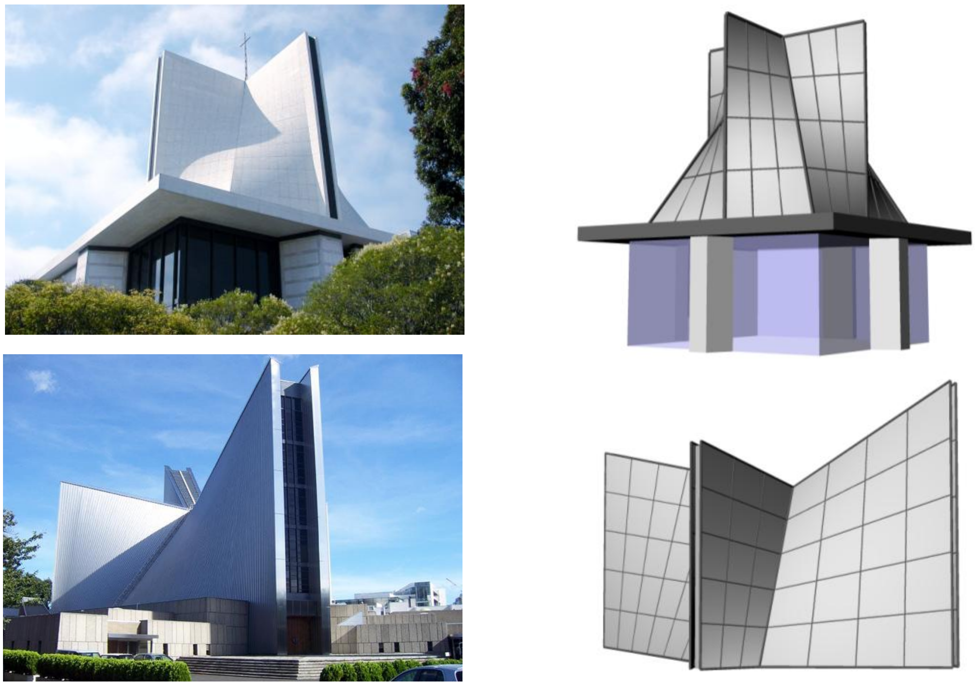

Figure 9. Hyperbolic paraboloids in use by architectural design. At the top St. Mary‘s Cathedral – San Francisco, USA, at the bottom St. Mary’s Cathedral - Tokyo, Japan.

Conclusion and future work

We discussed possible approaches how to increase the interest in studying geometry by using computers. The usage of 3D modeling we demonstrated on some concrete examples from the field of descriptive geometry mainly geometry of surfaces. If we can work with 3D model of the object in modeling software and move with it we understand more its properties.

The responses from students to using mathematical and modeling computer software (for example Rhino) in teaching geometry are very positive. Students are satisfied because the computer software is very motivational and attractive for them. Classical and descriptive geometry are more understandable and geometry in general becomes modern discipline.

We also have web pages with database of geometric tasks, images and 3D models (Surynkova, 2011) – the outputs of various software which are suitable teaching aid can be used in publications and also for e-learning not only for our students. In future work we will focus on further methods which can improve the teaching process. We plan to extend our gallery of 3D models and geometric tasks.

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