International Conference on Mathematics Textbook Research and Development 2014 (ICMT-2014)
29-31 July 2014, University of Southampton, UK

MODERN DESCRIPTIVE GEOMETRY SUPPORTED BY 3D COMPUTER MODELLING

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In my research, I investigate innovative methods of explaining complex concepts in descriptive geometry. These novel methods will be employed in my upcoming textbook on descriptive geometry for undergraduate students. The innovation in explanation and didactic methods will include 3D computer modelling and interactive software visualization. In this paper, I will present my recent advances in teaching aspects of several descriptive geometry topics – parallel and central projections, especially linear perspective or the geometry of curves and the geometry of surfaces in technical practice. For selected topics, I will provide examples of usage of 3D computer modelling. My aim is to stimulate students' interest in the study of geometry, motivate them, improve their understanding of geometry, innovate the methods of teaching geometry, achieve better results in examinations and put emphasis on practical use of geometry. I would like to attract students to the traditional topics of descriptive geometry by using modern methods. At the same time, since I would like to motivate students to think about concepts in descriptive geometry, the suggested teaching aids should not suppress independent thinking. In other words, these aids should not be designed so as to solve all questions for my students – some should be left for the students to address. Surfaces used in technical practice are very suitable for presentation by means of 3D printing. Therefore, I plan to model these surfaces in the 3D modelling computer software in cooperation with my students so that these computer models can subsequently be printed on a 3D printer to foster spatial imagination.
Keywords: descriptive geometry; 3D computer modelling; geometry of curves and surfaces; application of 3D printing.

INTRODUCTION AND MOTIVATION

Descriptive geometry is an area of classical geometry dealing with the representation of three-dimensional objects in two dimensions where 3D computer modelling and interactive software visualization can be applied with potentially significant impacts. Hence, the typical task in descriptive geometry is to represent three-dimensional objects on a two-dimensional display planar surface and to reconstruct 3D objects from the two-dimensional result of the projection. Descriptive geometry deals with those representations which are one-to-one correspondent. In order to gain deep understanding of descriptive geometry it is necessary to have knowledge of the fundamentals of geometry, the properties of geometrical objects in the plane and in the space, and their relations. This means that, in addition to geometrical projection, descriptive geometry should focus on special types of technically important curves and surfaces in engineering practice.

In general, geometry can be conceived as an independent discipline comprising various branches (descriptive, Euclidean, differential, algebraic, no-Euclidian, computational,

applied geometry and so on) and it also forms the basis for many modern applications. The motivation for studying geometry can be found in building practice, engineering and construction practice, architectural and industrial design, production industries, export of real interiors and exteriors into the virtual worlds of computer games, digitization of real objects by 3D scanning, digital surface reconstruction from point clouds, replication of the shapes of real objects using 3D printing, computer graphics and many more, (Pottmann et al., 2007). The common basis of all these modern applications is the combination of geometric principles and knowledge. Applied methods are often based on elementary geometry.

From a historical point of view, the development of descriptive geometry reached its greatest height in the last century. Nevertheless, even despite today's innovative approaches and continuous development of modern computer technology and equipment, descriptive geometry has not lost its importance. The role of descriptive geometry in practice is irreplaceable in such branches in which correct visualization is crucial. Owing to the fact that all of the mentioned application fields are dependent on clear illustrations and visualization, descriptive geometry has become the language of designers, engineers and architects. Overall, geometry in the plane and in the space, i.e. the properties of geometrical objects and their relations, form a part of many modern and contemporary scientific fields.

Geometry represents one of the highly demanding fields of mathematical science which require logical thinking and which also strongly stimulates spatial imagination. The study of geometry, and especially descriptive geometry, represents an ongoing challenge in terms of research and practice.

On the top level, the paper is organized into two parts. The first part explains possible novel methods of teaching descriptive geometry which include 3D computer modelling and interactive software visualization and presents new study materials and web support for descriptive geometry. This part is largely a summary of the existing concepts used in my lessons. The second part contains the main contribution of my work: the description of the upcoming textbook on descriptive geometry for undergraduate students. The conclusion is devoted to my future work and research in geometric fields.

3D COMPUTER MODELLING IN TEACHING AND STUDYING GEOMETRY

Descriptive geometry, and geometry in general, is a rather unpopular subject in secondary and undergraduate education due to its level of difficulty. It has been an unfortunate tendency in the recent years in the Czech Republic that the interest in studying this engaging mathematical discipline in on the decrease. In some cases, teachers of mathematics also show a lack of interest in teaching geometry. But primarily, it is necessary to offer education of geometry of a high quality and sufficient degree at elementary schools because the development of spatial imagination in early childhood is crucial.

The study of descriptive geometry includes, both at secondary schools and colleges, sketching and drawing activities. It may seem to be useless due to widespread availability of computers and modern software. Of course, we have to follow general trends and adapt teaching methods to the real practice, which, however, does not imply that classical drawing is outdated. We rely on these tools when developing of our initial ideas and finding solutions

to geometrical problems. Drawing and sketching helps us develop our precision skills and patience.

Through modernization of descriptive geometry to better results

Currently, computer aided design (*CAD*) is a common tool used in the process of designing, design documentation and construction for modelling and drawing, and generally throughout the entire design process, (Farin et al., 2002). There exist professional graphics software applications and environments which provide the required user input tools, and speed up production. Similar software can be used in teaching traditional geometric subjects, including descriptive geometry. As there is a wide range of inexpensive or free software applications for geometry and mathematics, it is not necessary to work with expensive CAD applications.

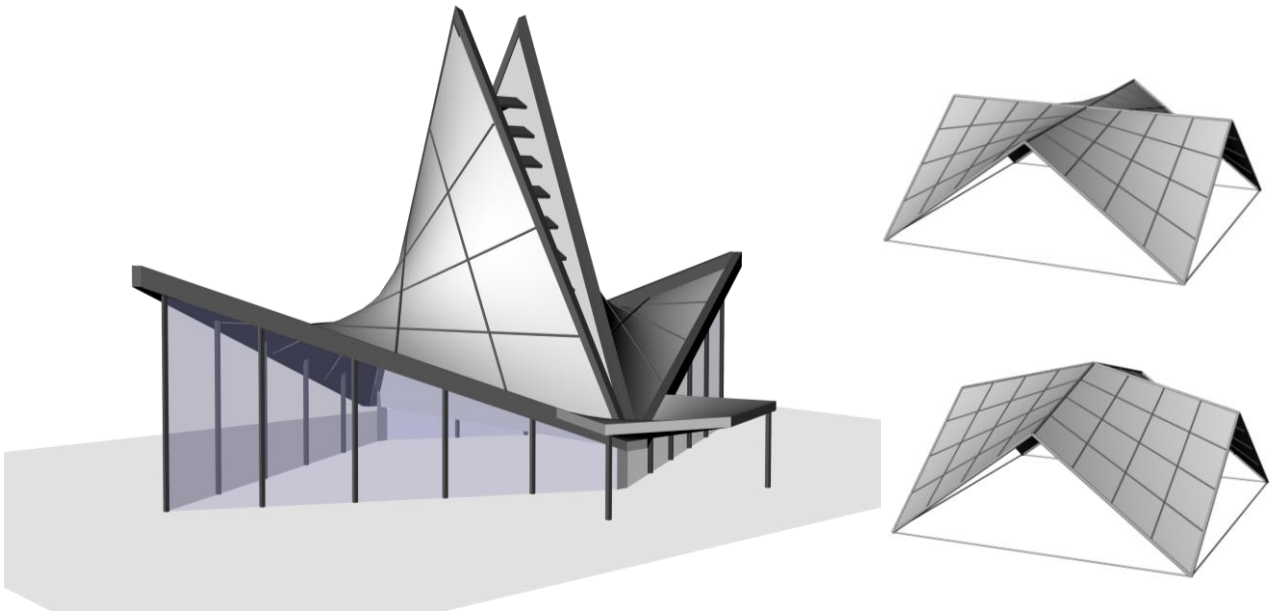

Figure 1. Example of 3D models created in Rhinoceros

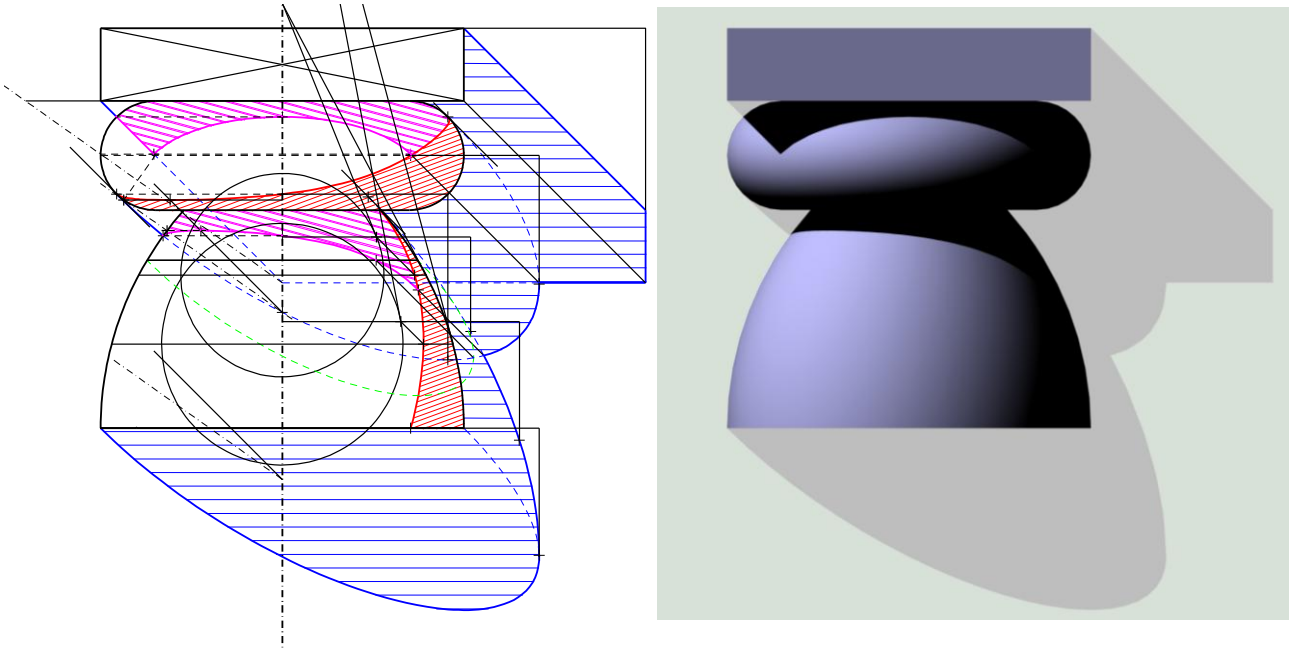

Figure 2. Examples of computer drawings created in Rhinoceros

In order to provide insight into more complex geometric problems and to increase the interest in geometry, I have integrated 3D computer modelling in my descriptive geometry lessons at the Faculty of Mathematics and Physics at Charles University in Prague. I work with the *Rhinoceros* (*NURBS Modelling for Windows*) software which is a commercial NURBS-based 3D modelling tool commonly used in the process of designing, design documentation and construction. I use Rhinoceros to create 3D models of geometric objects and situations in the space. It should also be noted that if we work with 3D modelling software, we can change the view of a designed object and see spatial geometric objects from another perspective which provides a clearer idea of the object. Examples of some 3D models are provided in Figure 1. We also use Rhinoceros to draw up constructions in the plane. As has already been pointed out, we do not intend to abandon traditional hand drawing methods because computer drafting is not efficient in developing our skill and thoroughness. Computer drafting is a modern auxiliary method which is also capable of yielding more precise results. Examples of computer drawings are shown in Figure 2.

The use of modelling and graphics software in teaching geometry increases students' interest in the subject and ensures their active involvement in the lessons, which is evident from the reactions of my students and also from their interest in these issues when dealing with their seminar projects or bachelor and master theses. 3D computer modelling is also an efficient aid in innovating the teaching of geometry and achieving better results.

I have been seeking to establish a stronger connection between descriptive geometry and its practical application and the extension of descriptive geometry with knowledge of computer graphics and computer geometry. The integration of descriptive geometry with 3D computer modeling appears to follow as a logical step.

Electronic study materials and web support for descriptive geometry

I have been gathering all of the aforementioned outputs obtained during the preparation of descriptive geometry lessons to create electronic collections of examples as well as for the purposes of new electronic methods of study of materials that relate to various geometric topics. All of these outputs are published on the website http://www.surynkova.info/, (Surynková, 2014). The site is continuously updated and it is intended not only for my students but also for everybody who is interested in geometry (some of the links are in English). New study materials and examples are dedicated to geometric constructions; there are also 3D computer models, examples of students' works and many more.

NEW TEXTBOOK ON DESCRIPTIVE GEOMETRY DESIGNED WITH 3D COMPUTER MODELING

This paper explicitly addresses the content and the design of a new printed textbook on descriptive geometry which I have been working on. This textbook is primarily dedicated to geometric topics such as curves and surfaces, solids, their definitions and properties, their parallel and central projections and the geometry of shadows. The textbook is intended mainly for students of the Faculty of Mathematics and Physics of Charles University in Prague and the first edition is planned to be published in Czech. The textbook will be illustrated using 3D computer modelling and modern software visualizations; I plan to use

exclusively the already finished outputs. The important part of the publication is the collection of examples with solutions and examples for testing purposes.

Case study for textbook chapters

Let us now focus on some parts of the planned chapters in the upcoming book, and describe its expected design. I am currently working on the theoretical aspects of special groups of surfaces used in engineering practice. The book will define each regular geometric surface and introduce its properties. Let us, for instance, look at an example of a part dedicated to helical surfaces; the concept of the chapter is as follows.

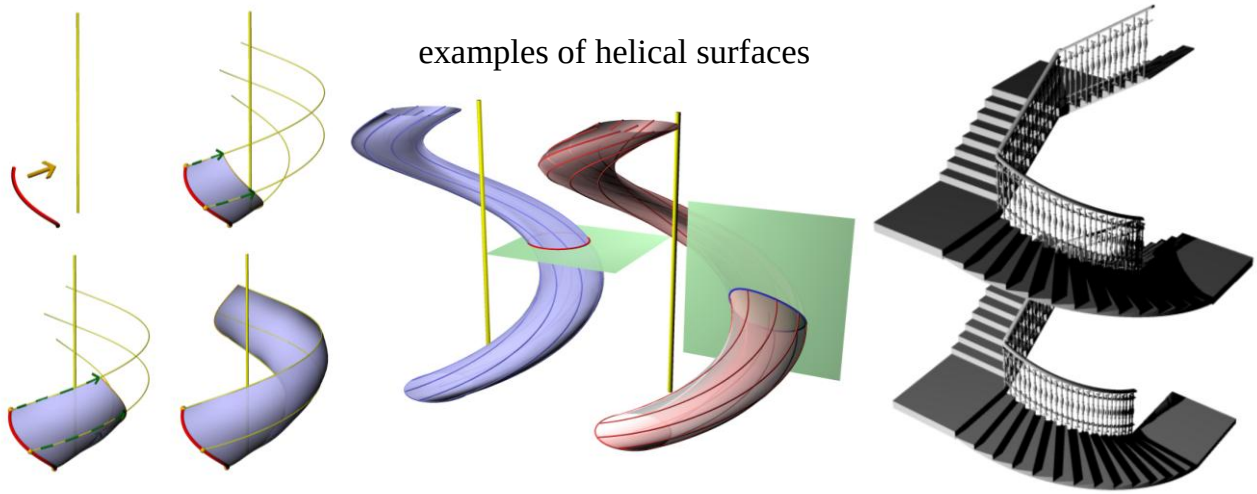

Figure 3. The illustration of helical surfaces

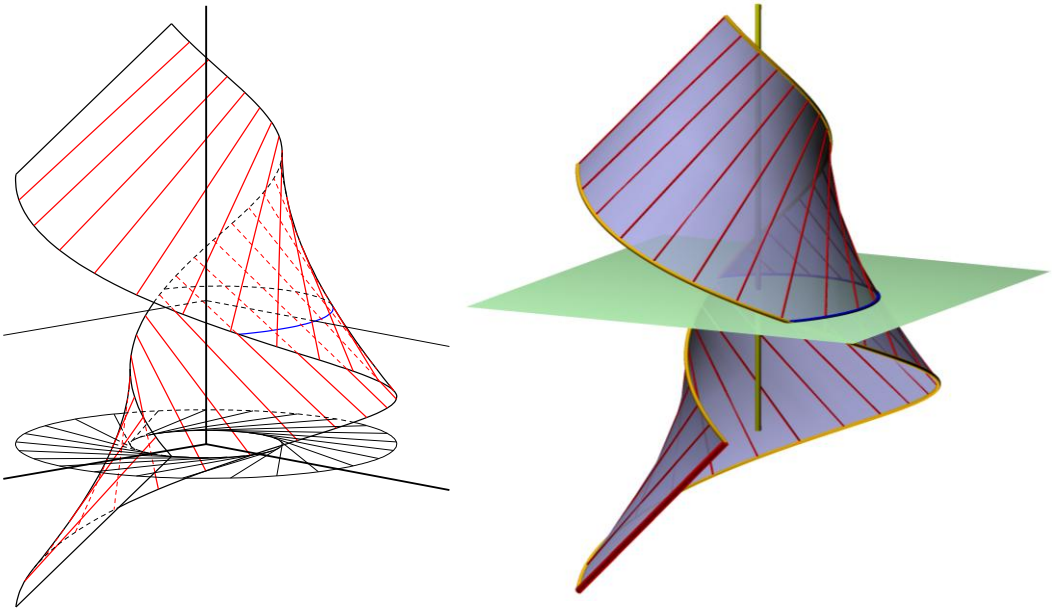

Figure 4. The result of the projection of helical surface and the situation in the space

First, a theoretical explication regarding the determination of helical surfaces is provided, accompanied with illustrations from the 3D computer modelling software. It is assumed that the source files of most pictures from the textbook are available on the attached removable

media to allow practical exercises regarding the properties of the discussed surface or spatial situations directly in the software. The illustration of helical surfaces with a brief description is shown in Figure 3.

The second part of each chapter is devoted to parallel and central projections of the studied surfaces, accompanied with a typical example including a detailed step-by-step solution and illustration. The typical task is to construct a parallel or perspective view (a two-dimensional image) of a particular surface. Figure 4 shows an orthogonal axonometric projection of a helical surface, defined by an axis and screwing segment line. The result of the projection and also the situation in the space are visible. Every illustration is made using 3D computer modelling.

The last part of every chapter comprises a collection of examples for exercising the properties of surfaces in various projections. Students can solve the tasks using 3D modelling or graphics software or they can draw the solutions by hand. When using software, it is necessary to construct the silhouette of the surface; if drawn by hand, the aim is to depict some of the important curves on the surface. In both cases, the result is a planar image.

An interesting additional feature of these examples is the possibility to model the surfaces in 3D modelling computer software in space. The spatial situation and principles of projection can also be demonstrated. The virtual model of the spatial situation and 3D virtual models of surfaces make a significant contribution to the development of spatial imagination. Some examples in the book are added in the form of 3D models on attached removable media and additional 3D computer models can be created in cooperation with my students, for example, as part of their theses.

CONCLUSION AND FUTURE WORK

Two areas are addressed in this paper – the possible *methods of innovation in teaching descriptive geometry* (including 3D computer modelling and the creation of new study materials and web support for descriptive geometry) and *the description of the upcoming textbook on descriptive geometry* for undergraduate students.

A survey of contemporary mainstream fields of application of geometry is provided. The main aim is to improve and innovate the methods of teaching descriptive geometry by using 3D computer modelling and enabling connection with practice. It is planned to integrate the suggested outputs from 3D computer modelling software into my new textbook on descriptive geometry. In the future, it is envisaged to publish the textbook in English translation. Some parts of the textbook are also planned to be published on the Internet.

References

Farin, G., Hoschek, J., Kim, M.-S. (2002). *Handbook of Computer Aided Geometric Design*. The Netherlands: Elsevier Science.

Pottmann, H., Asperl, A., Hofer, M., Kilian, A. (2007). *Architectural Geometry*. USA: Bentley Instute Press.

Surynková, P. (2014). *Academic website Petra Surynková*. http://www.surynkova.info/, June 2014.